model in the coflowing stream case is its inability to match the exponential decay rate of the jet. The experimental decay rate exponent is dependent upon the velocity ratio, while the Prandtl model exponent is equal to unity. The present modification improves the model by making the exponential decay rate dependent upon the initial velocity ratio. The resulting modified Prandtl eddy viscosity model is

$$\varepsilon = kb(u - u_e)[(U - u_e)/(u - u_e)]^m \tag{2}$$

Thus the Prandtl model is simply multiplied by the inverse of the quantity usually plotted as the ordinate of the velocity decay plots, raised to the power of the velocity ratio. In the case of the absence of a coflowing stream, the modified Prandtl model reduces identically to the classical Prandtl model.

The ability of the modified Prandtl model to predict the far field decay of coaxial jet is shown in Fig. 1. The new model successfully predicts the centerline velocity decay from approximately $\bar{x} = 20$ downstream.

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Base Pressure Distribution of a Cone at Hypersonic Speeds

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Nomenclature

M = freestream Mach number
p_B = base pressure
p_∞ = freestream static pressure
r = radial distance from the centerline
R = maximum base radius
Re = unit Reynolds number
α = angle of attack

Introduction

LITTLE or no data are available on base pressure distributions at high angles of attack in the hypersonic speed range. This is mainly due to the complex nature of the flowfield surrounding a three-dimensional body and the difficulties involved with measurements in the near wake. Therefore, the basic objective of the current work was to obtain reliable, interference-free base pressure distribution data on a 10° half-angle sharp, flat-based cone at high angles of attack (0 to 80°) and at hypersonic speeds ($M = 5.30$, 6.34, and 9.94) with varying Reynolds number ($Re = 3.0, 6.4, 11.2 \times 10^5$/ft). To this end, freeflying instrumented models were developed and tested in a freejet hypersonic facility. They were designed to be injected into the flowfield at predetermined angles of attack and roll and to be released with adequate vertical velocity to fly through the test section. The decision to use free-flying models was based on reported data on sting interference as well as Pick's more recent investigation.[1]

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Index categories: Jets, Wakes and Viscid-Inviscid Flow Interactions, Supersonic and Hypersonic flow.

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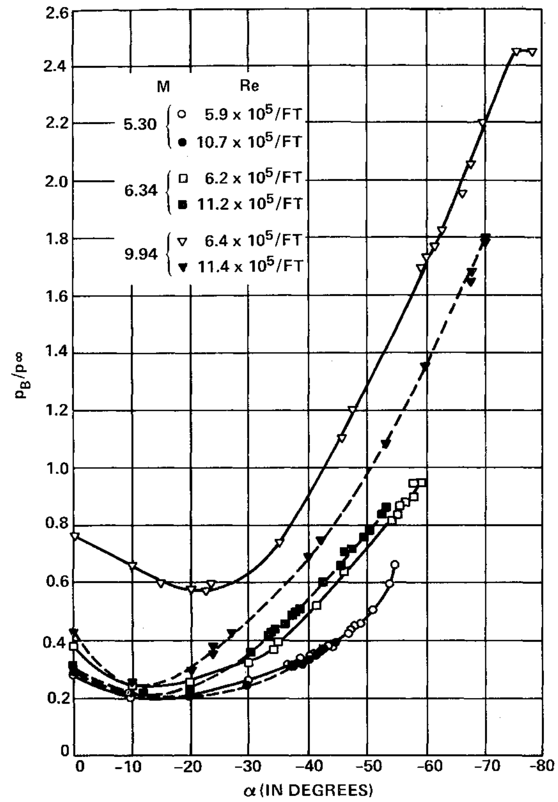

Fig. 1 Centerline base pressure ratio as function of angle of attack *M* and *Re*.

Description of Experimental Techniques

All experiments were conducted in the NAVSHIPRANDCEN variable Reynolds number hypersonic tunnel with a 13.5-in. open-jet test section. The 10° half-angle cone models used in the test program were 6 in. long and the outer shells were machined from stainless steel. A conical brass weight and threaded slugs, attached to the forward part of the model interior, provided for center-of-gravity adjustment. Four differential pressure transducer telemeters were the principal instrumentation and were located in the aft portion of the conical shell. Detailed performance characteristics and construction of these units are described by Harrison.[2] A mercury battery pack provided the power supply to the transducers. The transducer measuring ports were connected to the base plate at $r/R = 0$, 0.24, 0.47, and 0.71.

Signals from the telemetry units were intercepted by a folded dipole antenna that was completely outside the hypersonic stream and connected to a signal conditioning and processing network. The incoming data were recorded on a multichannel oscillograph. Two high-speed motion picture cameras recorded the motions of the model. Computer programs converted the information from the oscillograph records and high-speed motion pictures to base pressure ratios p_B/p_∞, angles of attack as functions of time. Optical lens distortions and human errors in data reduction were internally compensated for in the computations.

Prior to its injection into the hypersonic jet stream, the model was guided by a specially constructed drop mechanism that held it at a predetermined pitch angle. As the model was completely submerged into the inviscid core of the jet flow, the restraining arms opened and released the model. Within 10 msec from the initial release, the drop mechanism moved out of the flowfield to ensure disturbance-free conditions. The average available flight time, during which interference-free data were obtained,

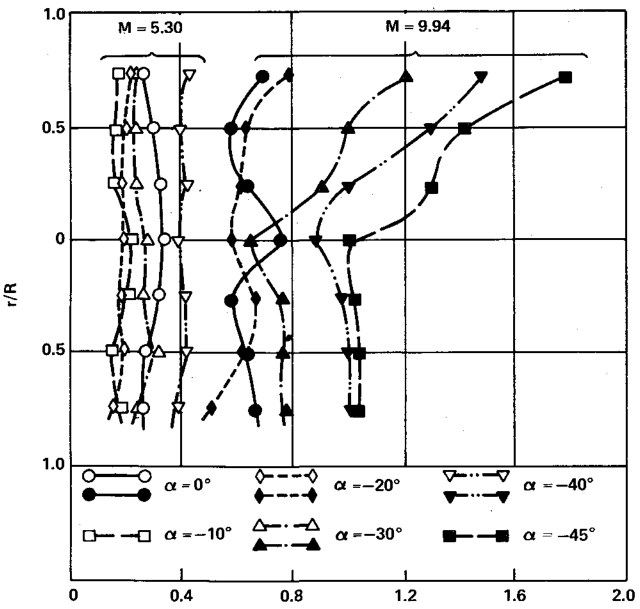

Fig. 2 Base pressure distribution in the vertical meridian plane at $Re \approx 6 \times 10^5$/ft.

was about 60 msec. All the model releasing and instrumentation recording functions were automatized using a multicam timer sequencer. For a given flow condition and angle of attack, a series of 12 drops at 30° roll-angle increments provided good definition of the complete base pressure distribution.

A pressure-time response study[3] indicated that true base pressure levels were measured within 5 msec after injection into the flow stream. The effect of close proximity on simultaneously operating transducer telemetry units did not produce errors higher than $\pm 1.3\%$.[4] Corrections to compensate for interaction errors were contained in the daily calibration and data reduction procedures. In all, the total error in the measured base pressure data due to instrumentation, interaction, and time-response errors was estimated to be within $\pm 5\%$.

Results and Discussion

Centerline base pressure ratio as a function of α, M and Re is shown in Fig. 1. Note that p_B/p_∞ reaches a minimum value at or around $\alpha = -10°$ and $-30°$; however, beyond $\alpha = -30°$, the base pressure ratio increases quite rapidly. In general, p_B/p_∞ decreases with decreasing Mach number, which is in accordance with published results elsewhere. Further examination of the results shows that for $M = 9.94$, increasing Reynolds number causes a decrease in p_B/p_∞ values at identical α conditions. The effect of Reynolds number was small for $M = 6.34$ and negligible for $M = 5.30$.

These results provide some indications as to the state of the cone boundary layer as well as the flow conditions immediately downstream of the base. Studies performed in a preliminary program indicated that the cone boundary layer and the base flow at $M = 9.94$ were laminar in the unit Reynolds number and angle of attack range tested. Both the preliminary and present results indicate that conditions at the base at $M = 6.34$ and 5.30 ranged from laminar at $\alpha = 0°$, to transitional below $\alpha = -20°$ and turbulent where $\alpha > -20°$. The reason for this phenomenon was the rapid decrease of transition Reynolds number with M and α, due to early instabilities caused by the crossflow formation and vortex shedding action. This is further substantiated by Figs. 2 and 3. Notice that for $M = 5.30$ and at $\alpha = 0°$, p_B/p_∞ is maximum at the centerline and tapers off toward the edges indicating laminar flow. However, when $\alpha > -20°$, the distribution is very nearly uniform in the entire base area (Fig. 3a). This indicates turbulent flow conditions, where the nonuniformity of the base pressure distributions decreases, due to large mixing. In contrast to this, at $M = 9.94$ the base pressure ratio is extremely nonuniform in the base area at high α where the flow conditions were determined to be laminar (Fig. 3b). The base pressure decreases from the windward generator toward the centerline where it reaches a minimum value and thereafter increases rather abruptly toward the leeward side where it is 70 to 80% higher than at centerline (Fig. 2). Indications are that the primary shock on the windward side deformed the inner shear layer at the base region causing local expansion and consequent drop in local pressure toward the centerline. On the other hand, local flow reattachment close to the leeward generator might be responsible for the increase of p_B/p_∞ from the centerline toward the leeward side. Many aspects of the observations of the current work were confirmed and reported by other investigators.

In summary, when laminar flow conditions prevail, the base pressure distribution is extremely nonuniform and the external flow quite complex. Whereas at turbulent flow conditions nearly uniform base pressure distribution exists even at high angles of attack.

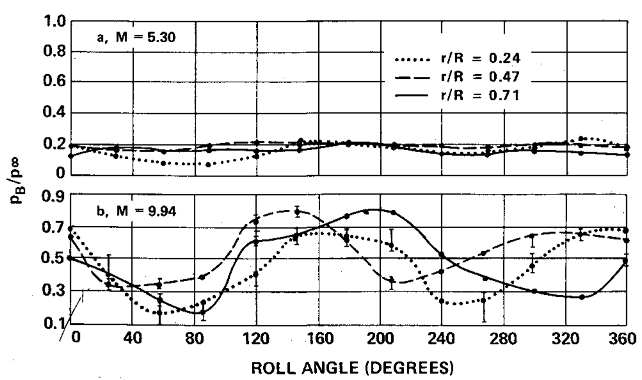

Fig. 3 Base pressure distribution at $\alpha = -20°$ and $Re \approx 6 \times 10^5$/ft.

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Placement of Observer Eigenvalues

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Introduction

WHEN linear modern control theory techniques are applied to practical control problems, one requirement which usually arises is knowledge of the system state vector; perfect

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